A Framework for Achieving the Essential Academic Learning Requirements in Mathematics

Grades 8-10

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State Superintendent of Public Instruction
A FRAMEWORK FOR ACHIEVING
THE ESSENTIAL ACADEMIC LEARNING
REQUIREMENTS
IN MATHEMATICS

Grades 8–10

June 9, 2000

This framework is designed to assist teachers in planning and implementing the mathematics curriculum. It provides a focus for assessment that emphasizes achievements in mathematics understanding and skills appropriate for students at middle and lower secondary levels. As such, it builds on the knowledge and skills developed in frameworks specifying the Essential Academic Learning Requirements for K-7. It also serves as a basis for documenting and reporting students' progress in mathematics understanding and skills to colleagues and parents.
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Introduction

This Framework has been developed to support the Essential Academic Learning Requirements of the State of Washington in the discipline of mathematics. Local mathematics guidelines developed from this Framework will illustrate the specific nature and expectations of each community. This framework is designed to provide a listing for teachers, curriculum developers, and the general public in what the necessary components of a contemporary program in mathematics entails. It provides guidelines for shaping, implementing, and assessing a curriculum that will prepare students for life, the workplace, and further study in mathematics. It also serves as a basis for framing reports of students' progress and checking progress in curricular activities across a school year.

Recognizing that there are many different curriculums and organizations of the subject matter material in the middle grades and secondary school years, districts should be aware that some license might be taken with the exact year-to-year placement of topics in this Framework with respect to the local curriculum framework. The local placement of topics probably would not differ more than one-year in the placement of topics from those presented here, with all topics being covered by the end of the tenth grade. The chart of each Suggested Assessment Evidence for the end of grades (eighth, ninth, and tenth) shows a listing of grade-appropriate activities and example outcomes corresponding to the Essential Academic Learning Requirements and other major recommendations for school mathematics. Further, specific forms of assessing such stages and forms of learning are suggested.

Teachers are reminded that the Framework offers suggested goals for each grade level. To assure students' continued progress in mathematics, the work of each grade level must be connected to that of previous grade levels. Content and skills from previous years must be revisited, practiced, extended, and applied as students move forward in the curriculum.

Problem Solving, Mathematical Reasoning, Communication, and Connections

Central to the goals of the mathematics curriculum and to the Essential Academic Learning Requirements in Mathematics are the processes of problem solving, mathematical reasoning, communicating, and connecting mathematical concepts and procedures in both everyday and technical ways. As such, these processes need to be infused in the curriculum in a consistent and constant fashion.

When students are engaged in meaningful mathematics, they work to solve and extend problems in appropriate and creative ways. Classroom activities should engage students in reasoning, communicating about, and connecting the mathematics studied to their daily experiences and to mathematics studied in earlier grades and to other fields of study. Communication about mathematical situations and their findings with their peers, their teachers, and their parents strengthens the connections and reinforces the worth of their work. This growth must be reflected in continued instruction which gives students rich opportunities to use and extend the problem solving strategies of the earlier grades.
and to employ a variety of representations (graphical, symbolic, numeric, and verbal). These efforts should be linked and validated through both inductive and informal deductive forms of reasoning (spatial, proportional, probabilistic, statistical, recursive, and structural). Throughout instruction and learning situations, students should have multiple opportunities to consider the history and uses of mathematics in ways that lift up the broad applicability of mathematics and its central role in the development of humankind.

The Framework is based on the assumption that the concepts and procedures in mathematics will be presented in a problem-solving setting. Students will be encouraged to encounter mathematics in settings that reflect the real-world, not artificial and contrived situations. This is not to say that mathematics cannot be presented in a pure and abstract form from time to time. Students need to see a balance between the "pure" and "applied" sides of the discipline. At the same time, curriculums and instructional programs need to bear in mind the importance of student motivation and the connection of the mathematics presented to the real world of the students and the applications they will make of the mathematics in their lives.

The assessment methods employed in evaluating student knowledge must also reflect a variety of techniques for evaluating student growth and progress in learning and applying the content of the curriculum. There must be a balance of approaches evaluating knowledge of facts, skill in applying procedures efficiently and effectively, productive problem-solving, correct reasoning, and communication of results. At times, these assessments call for quizzes and tests, but at other times, they call for various forms of student performance. Sometimes, this may take the form of writing in a journal, producing an explanation of a problem's solution, writing a proof of a statement, or providing a model of a given situation. Sometimes, teachers can assess these activities through observations, at other times through discussions or interviews, and on other occasions by posing an extension of the students' current study.

These techniques are detailed in the charts and described in the materials found at the end of each of the grade level portions of this framework. They are keyed to the various forms of assessment mentioned. In addition to assessing the students' progress in mastery of facts, concepts, principles, procedures, and processes, teachers need to be mindful about shaping students' conceptions about the nature of mathematics and both its role and potential power in their lives. Students must come to see mathematics as a strong, empowering force for opportunity and understanding in their lives, not as a gatekeeper that limits their choices.

"Quick Checks" provided at each grade level are designed to serve two purposes: (1) to identify mathematical content to be included in each grade level, and (2) to identify the critical thinking, problem-solving, connections, communication, and reasoning processes appropriate to that grade. Successful implementation of the Essential Academic Learning Requirements depends upon the degree to which all students are given the opportunity to develop the above described processes while participating in classroom learning that develops the concepts, principles, and procedures of mathematics.
EIGHTH GRADE

Content Overview: By the end of eighth grade, students can select and justify their choice for the type of number to use in a given situation, choosing from whole, rational, decimal, or integer. They can order and compare these numbers symbolically or figuratively on a number line or using symbols for equality and inequality. They can identify the properties for operations, including additive inverse and multiplicative inverse, for all decimal numbers. They can add, subtract, multiply, and divide with integers and combine integers with other forms of number in other computations. Students can use whole number exponents to discuss squares and cubes and discuss orders of magnitude for large numbers in scientific notation. They can find the least common multiple and greatest common divisor/factor for a pair of whole numbers. They understand the relationship between squaring a number and finding a number's square root. They can estimate the values of computations involving integers and other forms of numbers. They understand the impact of choice of unit on measurement and can find an approximation of the effect of error on a calculation involving measurements. Students develop and apply the measurement formulas for both surface area and volume of rectangular solids, prisms, pyramids, cylinders, cones, and spheres. In all of these areas, students should be able to represent the concepts and skills both concretely and with symbolic notation.

Students can describe, classify, and contrast geometric planar figures and solids. They can describe and sketch the cross section of a 3-dimensional figure cut by a plane. They can sketch a 2-dimensional representation of a 3-dimensional shape, from different points of view. They can use coordinate systems to display and investigate the relationships between lines in the plane. Students can use proportions to examine the relative measures of corresponding parts of similar figures and find other measures through the use of scaling with proportions or the Pythagorean theorem. These students are coming to understand the nature of geometry as a deductive system. Students can examine a simple game to determine if the outcomes are equally likely or if the game is unfair. They are able to make and study lists of possible outcomes for an experiment to determine probability in settings involving "and," "or," and "not." With data, these students are able to examine the mean, median, mode, and range for a set of univariate data and study the spread of the data through box-and-whisker plots. They can collect, record, organize, and analyze two variable data through scatter plots and informal trend lines. Students are able to translate among graphical, tabular, symbolic, and verbal expressions representing quantities using variables and expressions. They are able to use equivalent expressions to simplify expressions and solve linear equations. Students have an intuitive understanding of slope and its interpretation as a rate of change. As in number and operation areas, students should have the ability to represent and model these understandings in a concrete, as well as symbolic, way.
Process Overview: By the end of the eighth grade, students can solve one- and two-step problems using equations, translating patterns based on data, verbal explanations, or graphs into expressions or equations for solutions. Students are able to examine patterns related to simple open ended problems, selecting appropriate strategies, monitoring their progress, and making adjustments based on the feedback, abandoning or revising their approaches as necessary. They are able to conjecture and investigate their conjectures, using inductive methods of thought and justification. Students are able to develop explanations for their investigations and problem-solving activities, using manipulatives or appropriate technology for developing the explanation and illustrating the graphs, figures, or data involved. These students are growing in their ability to construct convincing arguments about statements formed from analysis of problems and other situations. Students should be able to summarize their approaches and give verbal presentations, as well as listen and ask questions of those presented by their peers. They are learning to compare and contrast mathematical concepts, as well as apply mathematics from their classes to the solution of problems in other disciplines. They are learning to gather information, organize and interpret it, and then choose a representation to use in communicating the data's meaning to others.
Content Quick Check

Does the student:

☐ Effectively and efficiently perform the operations for whole, rational, decimal, and integer computations?

☐ Select and choose the type of number needed for a given situation, judging whether exact answer or estimate is needed for a given situation?

☐ Compare and order symbolically or on a number line whole, rational, and decimal numbers, and integers?

☐ Correctly select the property of operations that justifies a given calculation or operation with numbers or variables representing numbers?

☐ Use exponential notation to represent and calculate whole number powers of numbers?

☐ Represent and interpret numbers using scientific notation?

☐ Find the least common multiple and greatest common divisor/factor for a pair of positive integers?

☐ Select appropriate units for the measurement of common situations involving length, area, volume, weight, capacity, and mass?

☐ Calculate the surface areas and volumes of common geometric solids (prisms, cylinders, pyramids, cones, spheres)?

☐ Visualize (both verbally and through a sketch) the planar cross section of a geometric solid in a given direction?

☐ Use a coordinate system to graph linear expressions and represent properties of lines (parallelism/perpendicularity)?

P Employ similarity and the Pythagorean theorem to find indirect measures?

☐ Represent relationships between corresponding parts of similar triangles and use proportions to find unknown measures in such triangles?

☐ Determine situations involving probabilities known as certain and impossible?
Evaluate the probability of a simple event using lists of outcomes?

Represent the central tendencies and spread of data using a variety of graphs, including box-and-whisker plots?

Describe and extend a numerical pattern?

Translate between equivalent forms of expressions and equations using basic properties of equality and operation?

Set up and solve one- and two-step linear equations representing real-life situations?

Translate representations easily between numerical, graphical, symbolic, and verbal forms in problem-solving situations?
Process Quick Check

Note: Problem solving, mathematical reasoning, communication, and connections are necessarily infused throughout the curriculum. Although the processes are quite similar for each grade level, implementation of them will vary based on the developmental level of the students. Specific examples appropriate for Grade 8 can be found on the Suggested Assessment Evidence for the End of Eighth Grade chart.

Does the student:

- Develop, use, and explain (orally and in writing) a variety of strategies and approaches (guess/check/revise, work backwards, draw a diagram, make a chart, use a formula, write an equation) in problem-solving situations?

- Reflectively monitor progress in problem-solving situations?

- Conjecture, investigate, and validate thinking in open-ended problem situations?

- Use manipulatives and appropriate technology in investigating and communicating mathematical work?

- Use appropriate and efficient approaches to finding numerical results (paper-and-pencil, mental mathematics, estimates or approximations, or technologically assisted)?

- Support his/her work with clear and mathematically correct reasoning?

- Communicate results in a clear and appropriate fashion?

- Correctly validate the reasoning used in arriving at results or establishing a statement?

- Summarize work in correct and precise ways?

- Read, write, listen, and discuss mathematics with others in appropriate ways?

- Recognize when to shift between different representations of mathematical concepts (graphical, numerical, symbolic, and verbal)?

- Recognize the extensive use of mathematics outside the classroom (in other disciplines, careers, etc.)?
### Suggested Assessment Evidence for the End of Eighth Grade

**CONCEPTS AND PROCEDURES**

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</table>

#### Number Sense

- Applies associative, commutative, identity, inverse, and distributive properties to simplify and complete rational number operations: 1.1 X X X X X
- Translates between simple fraction, mixed number, and improper fraction formats in representing and interpreting rational numbers: 1.1 X X X
- Represents relationships between whole, rational, integer, and decimal numbers on the number line: 1.1 X X X X
- Uses factors, multiples, and prime factorization to simplify and solve rational number computations (ex: writes 28/48 in reduced form): 1.1 X X X X
- Uses fraction, decimal, ratio, and percent equivalencies to solve problems (ex: finds percent equivalent to 7/8): 1.1 X X X
- Applies mental arithmetic to compute simple percentages (ex: 10%, 25%, 33.5%, 50%, 75%): 1.1 X X
- Writes decimal numbers using scientific notation (ex: 2,370,000 = 2.37 x 10^6; 0.035 = 3.5 x 10^{-2}): 1.1 X X
- Demonstrates proficiency in computation with positive and negative rational and decimal numbers: 1.1 X X X
- Creates and solves whole number proportions: 1.1 X X X
- Uses multiples of unit rate, or cost, to estimate the result of a specified operation or purchase: 1.1 X X X X X
- Gives estimates for values involving unit multiples using mental mathematics (ex: if 5 bottles cost $10, then 7 bottles cost $14): 1.1 X X X

#### Measurement

- Measures angles to the nearest degree with a protractor and estimates angle measurements to the nearest 10: 1.2 X X X
- Knows the number of degrees in a circle, triangle, and quadrilateral: 1.2 X X X
- Applies formulas for perimeter and area for triangles, standard quadrilaterals, and circles and surface area and volume for prisms, cylinders, and spheres: 1.2 X X X X
- Uses scales and ratios involving known measures to estimate or calculate measures of objects for which no direct information is given (ex: how much does a large can of juice cost if the small can costs 25¢, when both cans are shown): 1.2 X X X X
- Illustrates conversion between metric measures using powers of ten and movement of the decimal point (ex: 42.31 cm = 0.4231 m): 1.2 X X X

**Key for Assessment Processes**

1. Illustrated journals
2. Focused observationanecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests

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Suggested Assessment Evidence for the End of Eighth Grade (continued)

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<th>ASSESSMENT PROCESSES**</th>
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</tbody>
</table>

**Geometric Sense**

| Describes and classifies 3-dimensional figures using their defining attributes: faces, edges, angles, vertices, angle measures, and measures of faces | 1.3 | X | X | X | X |
| Recognizes, sets up, and interprets proportions between similar figures (ex: finds the length of a missing side in a triangle similar to a given one) | 1.3 | X | X | X |
| Describes and constructs simple transformations for complex figures using combinations of translations, reflections, and rotations | 1.3 | X | X | X | X |
| Models and sketches 2-D versions of 3-D figures and 3-D figures from 2-D views | 1.3 | X | X | X |
| Displays points and lines in the coordinate plane | 1.3 | X | X |

**Probability and Statistics**

| Discriminates between impossible and certain events, describing why they have the respective probabilities of 0 and 1 | 1.4 | X | X |
| Collects random samples and can describe the population it depicts | 1.4 | X | X | X |
| Lists the possible outcomes for a simple event/experiment and uses them to calculate the probabilities of various outcomes | 1.4 | X | X | X | X |
| Describes changes in a graph from one reporting point to the adjacent reporting point (ex: describes the growth in population in one decade versus the population in the next decade) | 1.4 | X | X | X | X |
| Recognizes the type of data involved in a situation, counts or measures, and chooses the appropriate type of graph to represent it (ex: describes a data set that requires a bar graph rather than a line graph) | 1.4 | X | X | X | X |
| Calculates and applies the mean, median, mode, and range for a set of data (ex: finds the average height and range of heights for a sample of students in a class) | 1.4 | X | X | X |

**Algebraic Sense**

| Describes and extends number patterns based on constant additions of the same term | 1.5 | X | X | X | X |
| Finds the value associated with a variable in a formula given values for the other variables in the formula | 1.5 | X | X | X |
| Finds the solution to a linear equation with integral coefficients (ex: solves 5x + 2 = 37) | 1.5 | X | X | X | X |
| Writes an equation representing a specified relationship between quantities (ex: what number when multiplied by 4 and then increased by 2 is 38) | 1.5 | X | X | X |
| Graphs inequalities on the number line (ex: shades all points on the line where x + 2 < 7) | 1.5 | X | X | X |

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<tbody>
<tr>
<td><strong>PROBLEM SOLVING</strong></td>
</tr>
<tr>
<td>Develops, uses, and applies (orally and in writing) strategies for solving multi-step problems (ex: determines the costs for a three year loan of $500 at 6% and a three year loan of $500 at 7.5%)</td>
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<tr>
<td>Determines the steps (orally and in writing) that the student used in solving a particular problem</td>
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<tr>
<td>Locates or identifies the data or information needed to construct a solution to a problem.</td>
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<tr>
<td>Designs and conducts open-ended experiments (ex: creates and conducts a probability experiment)</td>
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<td>Adjusts the nature of questions being asked in problem solving situations relative to preliminary results observed</td>
</tr>
<tr>
<td><strong>MATHEMATICAL REASONING</strong></td>
</tr>
<tr>
<td>Forms a conjecture based on data or a pattern and uses examples to check the reasonableness of a conjecture</td>
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<tr>
<td>Relates the meaning of a solution to the conditions of the original problem</td>
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<tr>
<td>Develops a convincing argument for a conjecture based on evidence or examples</td>
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<tr>
<td>Creates a counterexample to a given invalid statement or justification for a true statement</td>
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<tr>
<td><strong>COMMUNICATION</strong></td>
</tr>
<tr>
<td>Gathers information from sources related to a problem at hand, such as a formula representing the situation, locating the appropriate values to substitute into the formula, etc.</td>
</tr>
<tr>
<td>Organizes and represents the data or information related to a given problem in an appropriate fashion for study/use in determining patterns or a potential solution strategy for the problem (ex: makes a graph of data related to an event in social studies)</td>
</tr>
<tr>
<td>Communicates orally or in writing the meaning of the pattern or data resulting from a given problem (ex: describes the trend present in a series of data related to an investigation in the social studies)</td>
</tr>
</tbody>
</table>

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Suggested Assessment Evidence for the End of Eighth Grade (continued)

<table>
<thead>
<tr>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES**</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 2 1 3 1 4 1 5</td>
<td>All of the Assessment Processes <strong>may be</strong> used for gathering evidence</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**

1. Illustrated journals
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- **CONNECTIONS**
  - Relates the characteristics of mathematical concepts or algorithms to one another, such as factors to multiples or parallelograms to rhombi: 5.1
  - Uses mathematical modeling in other disciplines (ex: identities trends in population data): 5.2
  - Determines if two mathematical representations are equivalent: 5.1
  - Illustrates how mathematics can be used in at least one career area: 5.3
NINTH GRADE

Content Overview: By the end of the ninth grade, students can explain the meaning of operations on real numbers and how operations relate to one another. In particular, they understand the roles of the additive and multiplicative inverse properties. They understand the properties of inequalities and the laws for equal addition and multiplication for equations. These students have an intuitive understanding of the density of real numbers and the distinction between rational and irrational numbers, with a special emphasis on pi and simple square roots. They have a grasp of the basic exponent properties including negative integral powers of a positive number. These students can distinguish between direct and inverse variation and describe the nature of each in terms of the types of graphs (linear and curved) related to such relationships. The students can both compute and estimate the values of indicated operations efficiently and accurately.

Students extend their understanding of the representing of lines on coordinate planes through understanding of the role of slope and the determination of parallel and perpendicular lines. They use coordinate grids to investigate and establish properties of geometric transformations, including magnifications. Students use line plots and histograms to examine the nature of data distributions, looking for spread and outliers. They recognize the relationship between variables in two-variable settings and discuss it in terms of trend and relationship. Students can find the equations of trend lines, and discuss their rates of change. They can interpret situations and represent and solve them through systems of equations, discussing the meaning of their solutions.

Process Overview: By the end of the ninth grade, students can formulate, represent, and abstract patterns from numerical and geometric situations, generalize them into algebraic expressions, manipulate these expressions to find equivalent expressions or solve equations, and communicate the results to their peers in writing, speaking, and graphical forms. They should be able to explain their reasoning, using organized lists and expressions developed from differences in T-tables based on successive integer inputs. They should be able to determine if a solution is unique or whether it is one of a list of possible solutions. They should be able to provide probabilistic, statistical, or combinatorial reasoning to back up inductive and elementary deductive reasoning for a statement. They should be able to shift between representations in explaining their solutions to problems and in describing examples of a mathematical concept or principle. They should be able to use spreadsheets, word-processing programs, geometric software, and statistical packages to illustrate and record their work. Students should be able to talk and write about the role mathematics plays in modern society and everyday events.
NINTH GRADE

Content Quick Check

Does the student:

- Compare and contrast the structure (operations, properties, including density) for the real number system and its subsystems (whole, integer, rational, and real)?

- Identify and use appropriately the additive and multiplicative inverses of numbers and variables in solving problems?

- Recognize and apply the properties of equality and inequality for equations and inequalities?

- Solve linear equations and inequalities, justifying steps when required?

- Compare and contrast the nature of an irrational number (notably small square roots for whole numbers and pi) with the nature of a rational number?

- Evaluate, correctly, expressions involving integer exponents, citing properties where necessary?

- Compute accurately and efficiently, using appropriate methods (paper-and-pencil, mental arithmetic, technology) with varied number systems and expressions?

- Estimate when appropriate and with the required level of accuracy?

- Identify the nature and exact value of the slope of a line graphed on a coordinate plane and give its symbolic form?

- Relate the nature of parallel and perpendicular lines in spatial settings with their algebraic representations on a coordinate plane?

- Use line plots, histograms, and other forms of graphics to investigate distributions of data, noting outliers, spread, and symmetry.

- Express the slope (rate of change) for a line graphed in the coordinate plane and identify its intercept(s)?

- Represent lines symbolically or graphically, shifting from given data, graphs, or symbolic forms?
Evaluate linear and quadratic expressions for given values?

Relate the solutions of linear and systems of linear equations, to points on their graphs?

Translate a given situation into a system of equations, represent them, and solve them to resolve the original situation?

Recognize the relationship between expressions, relations, and equations at an elementary level (data and graphs)?
Process Quick Check

Note: Problem solving, mathematical reasoning, communication, and connections are necessarily infused throughout the curriculum. Although the processes are quite similar for each grade level, implementation of them will vary based on the developmental level of the students. Specific examples appropriate for Grade 9 can be found on the Suggested Assessment Evidence for the End of Ninth Grade chart.

Does the student:

☐ Recognize, formulate, solve, perform operations, and communicate effectively in problem-solving situations:

☐ Abstract patterns from tabular data and generalize them in algebraic expression form?

☐ Select appropriate forms for communicating problem solution reports depending on audience and level of detail required?

☐ Employ problem-solving strategies to assist in identifying patterns in problem situations?

☐ Provide convincing arguments for conjectures based on inductive and informal deductive methods?

☐ Translate between representations (numerical, graphical, symbolic, and verbal) as appropriate in gathering and recording information, in problem solving, and in communicating results?

☐ Employ spreadsheets, word-processing, graphing programs, statistical packages, geometry software and other information processing aids in testing conjectures, solving problems, and communicating results?

☐ Talk and write effectively in describing his/her problem solving and project work in mathematics?

☐ Recognize the pervasive role mathematics plays in modern society?
## Suggested Assessment Evidence for the End of Ninth Grade

### CONCEPTS AND PROCEDURES

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<th>EALRs</th>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applies associative, commutative, identity, inverse, and distributive properties to simplify and complete decimal number operations</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Uses properties of real numbers and order in interpreting rational number situations (ex: finds a rational number between 314 and 718)</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Orders sets of rational numbers, integers, or real numbers relative to their values in abstract or concrete settings</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Finds the absolute value of integers</td>
<td>1.1</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Expresses whole numbers in prime factored form using exponential notation, such as 48 = 2^4 \cdot 3</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Converts between common infinite repeating decimal representations and their rational number representations (ex: finds the fraction equivalent to 2.6666...)</td>
<td>1.1</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Describes the nature of an irrational number and provides examples like pi and the square root of 2</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Understands exponent properties including negative integral powers of a positive number</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Recognizes and interprets numbers in scientific notation in various formats such as standard form, scientific notation, or calculator format (3.5E-6)</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Computes in situations involving rational numbers, decimals, integers, and real numbers</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Determines the reasonableness of a calculation involving rational or decimal numbers (ex: is it reasonable that a fast food restaurant sold $150,000 of hamburgers in a day)</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Differentiates between situations where estimates are sufficient and those for which exact values are required</td>
<td>1.1</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**

1. Illustrated journals
2. Focused observationianecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests

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### Suggested Assessment Evidence for the End of Ninth Grade

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**Measurement**

| Measures specified objects to the nearest linear unit given the appropriate measurement tool and uses the measure to compute the perimeter, area, or volume of triangles, standard forms of quadrilaterals, circles, prisms, cylinders, and spheres | 1.2 | X  | X  |
| Uses subdivisions of a figure to find or estimate the measure of a complex figure (ex: finds the area or perimeter of a figure with many parts, each of which can be computed or estimated separately) | 1.2 | X  | X  | X |
| Identifies and describes meaning of the slope of a line as a rate of change | 1.2 | X  | X  | X |
| Converts among measures for length, area, volume, mass/weight, or capacity within the U.S. Customary and within the metric system of measurement | 1.2 | X  | X  | X |
| Determines the precision and accuracy of measurements obtained with a given common measurement tool | 1.2 | X  | X  | X |

**Geometric Sense**

| Investigate properties of figures using measurement and coordinate grid representations, such as the diagonals of a parallelogram bisect each other and are perpendicular at the point of intersection | 1.3 | X  | X  | X |
| Constructs models of similar figures, showing the relationship between corresponding parts | 1.3 | X  | X  | X |
| Solves for measures of sides of similar figures using proportions | 1.3 | X  | X  |
| Uses proportions to find the sizes of sides of a figure that is magnified or reduced in size | 1.3 | X  | X  | X |
| Describes position of the image of a figure under a specified transformation (reflection, rotation, translation) on a coordinate grid | 1.3 | X  | X  | X |

**Key for Assessment Processes**

1. Illustrated journals
2. Focused observation/ anecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests
### Suggested Assessment Evidence for the End of Ninth Grade (continued)

<table>
<thead>
<tr>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>Probability and Statistics</strong></td>
<td></td>
</tr>
<tr>
<td>Gives the probability associated with a simple event</td>
<td>1.4</td>
</tr>
<tr>
<td>Identifies equally likely events (ex: rolling an even or odd number on a die)</td>
<td>1.4</td>
</tr>
<tr>
<td>Lists the outcomes for a two-stage event (ex: roll die or spin a spinner) and gives associated probabilities</td>
<td>1.4</td>
</tr>
<tr>
<td>Determines the probability of successive events like flipping a coin twice and getting two heads</td>
<td>1.4</td>
</tr>
<tr>
<td>Develops a conclusion about trends in data from examining a graph, such as explaining the rate of growth in a population</td>
<td>1.4</td>
</tr>
<tr>
<td>Uses scatter plots and box-and-whisker graphs to illustrate and describe the variability in data sets</td>
<td>1.4</td>
</tr>
<tr>
<td>Describes the range and the spread of a set of data</td>
<td>1.4</td>
</tr>
<tr>
<td>Analyzes data sets related to a given situation and writes a short description of the patterns observed, forming and defending the generalizations related to the patterns</td>
<td>1.4</td>
</tr>
<tr>
<td>Draws a trend line describing a set of data</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Algebraic Sense</strong></td>
<td></td>
</tr>
<tr>
<td>Represents numerical patterns, based on constant additions or multiplication, by extending them and giving an explicit expression involving a variable for the general term of the pattern</td>
<td>1.5</td>
</tr>
<tr>
<td>Describes and extends geometric patterns based on rotations or reflections of a geometric pattern</td>
<td>1.5</td>
</tr>
<tr>
<td>Evaluates simple expressions and equations via graphs, tables of values, or geometric relationships</td>
<td>1.5</td>
</tr>
<tr>
<td>Finds the solution to a linear equation using either symbolic or geometric methods (ex: finding the point where the graph of the equation crosses the x-axis)</td>
<td>1.5</td>
</tr>
<tr>
<td>Given the graph of a line, the student finds the equation of the line (ex: given a graph of a line crossing the x-axis at 3 and the y-axis at 5, the student finds its equation)</td>
<td>1.5</td>
</tr>
<tr>
<td>Uses rates and proportions to interpolate or extrapolate values on a graph (ex: finds the value associated with a point on a line graph between two given points or slightly beyond two given points)</td>
<td>1.5</td>
</tr>
<tr>
<td>Graphs inequalities in the plane [ex: finds all points (x,y) on a coordinate grid such that y &lt; 2x — 5]</td>
<td>1.5</td>
</tr>
<tr>
<td>Estimates the solution to a system of linear equations using graphs or a calculator</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**

1. Illustrated journals
2. Focused observation/anecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests

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Suggested Assessment Evidence for the End of Ninth Grade (continued)

<table>
<thead>
<tr>
<th>PROBLEM SOLVING</th>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translates conditions from problems into appropriate representations (ex: writes an equation representing the conditions for use in solving the problem)</td>
<td>2.2</td>
<td><strong>All of the Assessment Processes may be used for gathering evidence</strong></td>
</tr>
<tr>
<td>Identities the steps that he/she used in solving a problem (ex: found the right formula, substituted in the formula, solved the resulting equation, and interpreted the solution)</td>
<td>2.1, 2.2</td>
<td></td>
</tr>
<tr>
<td>Estimates what a reasonable result might be for a problem, selects an appropriate strategy, and uses the estimate to evaluate the resulting solution</td>
<td>2.1, 2.2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MATHEMATICAL REASONING</th>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explains the relevance of data to the problem at hand or whether an argument/statement is consistent with other information in a problem</td>
<td>3.1, 3.2, 3.3</td>
<td></td>
</tr>
<tr>
<td>Develops a convincing argument for the validity of a statement provided as an answer to a question</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>Develops and supports conjectures based on patterns, statements, or evidence related to a given situation</td>
<td>3.2, 3.3</td>
<td></td>
</tr>
<tr>
<td>Lists special cases related to a given problem and develops a conjecture based on the evidence in the cases</td>
<td>3.1, 3.2</td>
<td></td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**
1. Illustrated journals
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### Suggested Assessment Evidence for the End of Ninth Grade (continued)

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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>COMMUNICATION</strong></td>
<td></td>
</tr>
<tr>
<td>Develops an organized oral or written procedure or plan for communicating information relating to a problem and its solution to others</td>
<td>4.1</td>
</tr>
<tr>
<td>Expresses mathematical ideas verbally or in writing in a clear, logical, and correct fashion in discussing the solution, or progress toward a solution, to a problem</td>
<td>4.2</td>
</tr>
<tr>
<td>Represents information in a table, graph, or expression and translates between these forms of representation.</td>
<td>4.3</td>
</tr>
<tr>
<td>Chooses an appropriate form of representation for discussing a problem and its solution (verbal, tabular, graphical, or symbolical)</td>
<td>4.1, 4.2, 4.3</td>
</tr>
<tr>
<td><strong>CONNECTIONS</strong></td>
<td></td>
</tr>
<tr>
<td>Describes the relationship between algebraic and geometric representations of a concept (ex: describes why parallel lines have the same slope)</td>
<td>5.1</td>
</tr>
<tr>
<td>Outlines the relationship between mathematical concepts using words or diagrams (ex: describes with words and diagrams why the slopes of perpendicular lines are negative reciprocals of one another)</td>
<td>5.1</td>
</tr>
<tr>
<td>Uses mathematics to develop a model for a situation in another subject matter area and explains the relevance of the model (ex: explains with a graph that the volume occupied by a gas increases as the gas is heated)</td>
<td>5.2</td>
</tr>
<tr>
<td>Describes uses of mathematics in at least one career</td>
<td>5.3</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**

1. Illustrated journals
2. Focused observation/ anecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests

All of the Assessment Processes **may be used for gathering evidence**
TENTH GRADE

Content Overview: By the end of the tenth grade, students will demonstrate an understanding of the operations, properties, and algorithms associated with the real numbers and all of their subsystems. They will be able to efficiently apply this knowledge in situations involving either equalities or inequalities. Further, they can compare and contrast the various features of the whole numbers, integers, rational numbers, and real numbers. The students can extend a given arithmetic or geometric sequences or series given their defining expressions, find a specified term in such a sequence or partial sum of a series, or give an explicit rule for the nth term. They can relate the effects of change in linear units in measurement to corresponding changes in measurements of surface area and volume of figures. The students can develop and solve linear equations and inequalities to represent problems involving comparisons of quantities. In particular, they can use the concept of slope as a rate of change and use it to determine whether a given set of data represents direct or inverse variation.

Students are able to compose and decompose geometric figures in solving problems. They can explain the nature of a deductive system and its role in defining, conjecturing, and proving relationships. Students recognize and apply the major geometric relationships of lines, angles, triangles, quadrilaterals, regular polygons, and solids. They are able to represent spatial relationships using diagrams, graphs, or pictures. Students are able to provide convincing arguments of basic geometric relationships in a variety of forms. They can form conjectures about geometric relationships based on experiments with dynamic geometric software or work with physical models, as well as based on axiom systems. Students can represent probabilistic conjectures with algebraic or geometric models and use the models to find both simple and conditional probabilities. Students can design an experiment to approximate the probability of an event that is not easy to approach theoretically.

Process Overview: By the end of tenth grade, students should have developed a strong appreciation for the role of structure and verification as part of a mathematical system (undefined terms, defined terms, basic assumptions, choice of logic, and extensions of these through investigation and proof). They should be able to plan a solution strategy for a problem and efficiently monitor their progress and make alterations when necessary. Further, they should be able to pose extensions of problems they solve and apply their results in new settings. They should be able to use technology to retrieve and use information related to problems they are addressing and use that same technology to communicate their results. They should be able to evaluate the mathematical arguments of others, determining the validity or invalidity of their arguments. They should be adept at communicating mathematics to others, as well as in listening to others. In particular they should be aware of the role of scale in both algebraic and geometric settings, noting when it has been used appropriately and when it has been used inappropriately. Students should be able to outline the major aspects of algebraic and geometric reasoning and explain to some degree the history of each subject, noting the roles of the various individuals and cultures involved.
TENTH GRADE

Content Quick Check

Does the student:

- Employ the structure of the real number system in simplifying and carrying out operations and in translating statements to equivalent statements?
- Construct and solve linear equations, as well as systems of linear equations?
- Construct and solve linear inequalities?
- Recognize and extend arithmetic and geometric sequences, giving the values of specified terms?
- Recognize the effects of scale changes, changes in linear measurements, or changes in perspective on the measures and appearances of geometric figures?
- Form and use derived measures based on ratios of different measures (km/hr) or multiplication of measures (foot—pounds)?
- Estimate the amount of change related to a change in ratio or a change in input values for a formula?
- Compose and decompose geometric figures to investigate properties and construct proofs?
- Describe the nature of a mathematical system (undefined and defined terms, basic assumptions, choice of logic, and proven statements) and its role in shaping algebra and geometry?
- Represent spatial situations in 2- and 3-dimensional settings to show the relationships between objects and figures?
- Explain geometric transformations in terms of the movements and show the effects of successive geometric transformations?
- Form conjectures about geometric relationships and investigate conjectures using physical models, sketches, software, and inductive and deductive reasoning?
- Represent probabilistic conjectures using physical materials or geometric models and find or approximate probabilities for specified events?
Perform experiments related to a specified event and relate the differences that might occur between that probability and the theoretical probability for the same event?

Establish proportional models for geometric relationships and solve these models algebraically, interpreting the results back in the original geometric setting?
Process Quick Check

Note: Problem solving, mathematical reasoning, communication, and connections are necessarily infused throughout the curriculum. Although the processes are quite similar for each grade level, implementation of them will vary based on the developmental level of the students. Specific examples appropriate for Grade 10 can be found on the Suggested Assessment Evidence for the End of Tenth Grade chart.

Does the student:

☐ Monitor work in problem-solving situations, making alterations or changing lines of approach when needed?

☐ Pose extensions to problems or statements about a mathematical situation?

☐ Persist in attacking a problem as a personal challenge?

☐ Use technology in appropriate ways to investigate conjectures and to collect evidence for writing a proof or explanation of the situation?

☐ Evaluate the validity of statements in a careful and complete fashion?

☐ Listen carefully to mathematical explanations and reasoning and act on that information?

☐ Examine the connections between new concepts and principles in algebra and geometry, looking at ways of viewing the new knowledge in either form?

☐ Recognize the major events in the history of school geometry and school algebra, noting the individuals and cultures involved?
Suggested Assessment Evidence for the End of Tenth Grade

<table>
<thead>
<tr>
<th>CONCEPTS AND PROCEDURES</th>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Number Sense</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Applies associative, commutative, identity, inverse, and distributive properties to simplify and complete real number operations</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Demonstrates applications of the associative, commutative, identity, inverse, distributive, and order of operations properties with both real number and symbols (ex: simplifies $3(-x^2 - (x + 2))$)</td>
<td>1.1, 1.5</td>
<td>X</td>
</tr>
<tr>
<td>Uses factors and multiples of integers to determine divisibility, including greatest common divisor, and to find multiples, including least common multiples</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Finds common equivalents between fractions, decimals, and percents and between decimals and scientific notation</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Identifies numerical patterns based on constant additions (arithmetic) or constant multiplication (geometric) and finds general terms in explicit form for them and extends the patterns</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Uses ratios and proportions to represent quantities and solve problems such as those involving scale, rates, or percentages</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Computes with real numbers, integral powers, and square and cube roots in arithmetic or geometric situations (ex: finds the length of the diagonal of a rectangular solid)</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Uses mental arithmetic, paper-and-pencil methods, or technology (calculator or computer) to solve calculation problems involving rational, integer, and real numbers</td>
<td>1.1</td>
<td>X</td>
</tr>
<tr>
<td>Applies estimation techniques to determine the reasonableness of computational results or other estimates</td>
<td>1.1</td>
<td>X</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**
1. Illustrated journals
2. Focused observationianecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests
### Suggested Assessment Evidence for the End of Tenth Grade (continued)

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Measurement</strong></td>
<td></td>
</tr>
<tr>
<td>Measures specified objects directly using rulers, scales, containers, etc. and indirectly through the use of proportions and multiples</td>
<td>1.2</td>
</tr>
<tr>
<td>Demonstrates how changes in dimension of an object affect its perimeter, area, surface area, and volume (e.g. changes in area or volume when the length is halved)</td>
<td>1.2</td>
</tr>
<tr>
<td>Uses rates and proportions to calculate and to estimate the measures of objects commonly found in a school setting (e.g. height of gym wall, volume of cafeteria in m³, length of hallway, area of a stage)</td>
<td>1.2</td>
</tr>
<tr>
<td>Uses both the metric and U.S. systems of measurement to obtain specified measures of length, area, volume, mass capacity, and temperature</td>
<td>1.2</td>
</tr>
<tr>
<td>Justifies the selection of a specified unit of measure to perform a specified measurement task</td>
<td>1.2</td>
</tr>
<tr>
<td>Explains the relationship between the precision of a measurement and the overall accuracy of the measurement performed</td>
<td>1.2</td>
</tr>
<tr>
<td>Converts among different monetary units and related measurement systems</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>Geometric Sense</strong></td>
<td></td>
</tr>
<tr>
<td>Compares 2- and 3-dimensional figures in terms of their characteristics, such as faces, edges, and vertices</td>
<td>1.3</td>
</tr>
<tr>
<td>Analyzes and describes geometric relationships in terms of parallel, perpendicular, collinear, congruent and similar</td>
<td>1.3</td>
</tr>
<tr>
<td>Identifies and uses parallel and perpendicular lines on a coordinate plane</td>
<td>1.3</td>
</tr>
<tr>
<td>Creates scale drawings of specified figures given a scale and appropriate tools</td>
<td>1.3</td>
</tr>
<tr>
<td>Graphs and describes properties of figures on a coordinate grid</td>
<td>1.3</td>
</tr>
<tr>
<td>Distinguishes between properties of figures that have planes for faces and those that have curved surfaces, such as boxes and spheres, or prisms and cylinders</td>
<td>1.3</td>
</tr>
<tr>
<td>Compares/contrasts the properties of geometric figures in terms of symmetry, similarity, and congruence</td>
<td>1.3</td>
</tr>
<tr>
<td>Performs successive geometric transformations (translations, reflections, and rotations) on a specified figure in all four quadrants</td>
<td>1.3</td>
</tr>
<tr>
<td>Constructs specific geometric figures with compass and straightedge, protractor, or computer software</td>
<td>1.3</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**

1. Illustrated journals
2. **Focused** observation/anecdotal records
3. Individual interviews
4. Performance assessment
5. Traditional paper-and-pencil tests
## Suggested Assessment Evidence for the End of Tenth Grade (continued)

<table>
<thead>
<tr>
<th>Probability and Statistics</th>
<th>LINKS TO EALRs</th>
<th>ASSESSMENT PROCESSES**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constructs geometric models to find or estimate probabilities</td>
<td>1.4</td>
<td>X X X X X</td>
</tr>
<tr>
<td>Designs and carries out a simulation to find the probability of an occurrence of an event</td>
<td>1.4</td>
<td>X X X</td>
</tr>
<tr>
<td>Distinguishes between estimates of probabilities and actual exact values of the probability of an event</td>
<td>1.4</td>
<td>X X X X</td>
</tr>
<tr>
<td>Uses the fundamental counting rule to find the number of ways an event can occur</td>
<td>1.4</td>
<td>X X X</td>
</tr>
<tr>
<td>Discriminates in complex settings whether the outcome of a first event affects the probability of a later event (ex: in rolling a number cube and then flipping a coin, does the outcome of the roll affect the nature of the flip, i.e. drawing with and without replacement)</td>
<td>1.4</td>
<td>X X X X X</td>
</tr>
<tr>
<td>Uses knowledge of probability to find the probability of repeated events (ex: finds the probability of rolling six successive 5's with a fair number cube)</td>
<td>1.4</td>
<td>X X</td>
</tr>
<tr>
<td>Applies concepts of statistics to support a given argument in a term paper or debate</td>
<td>1.4</td>
<td>X X X X X</td>
</tr>
<tr>
<td>Collects data related to a specified question, describing how they should be collected and recorded</td>
<td>1.4</td>
<td>X X</td>
</tr>
<tr>
<td>Organizes and displays data in a form appropriate to the questions to which they will be applied, including histograms, pictographs, line and circle graphs, tables, stem-and-leaf plots, box-and-whisker plots, and scatter plots</td>
<td>1.4</td>
<td>X X X</td>
</tr>
<tr>
<td>Compares or contrasts two sets of data in terms of their means, medians, modes, range, and outliers</td>
<td>1.4</td>
<td>X X X X X</td>
</tr>
<tr>
<td>Draws supportable inferences from a given set of data, citing the basis for the generalizations drawn</td>
<td>1.4</td>
<td>X X X</td>
</tr>
</tbody>
</table>

**Key for Assessment Processes**
1. Illustrated journals
2. Focused observationianecdotal records
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5. Traditional paper-and-pencil tests
**Suggested Assessment Evidence for the End of Tenth Grade (continued)**

<table>
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<tr>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

**Algebraic Sense**

- Recognizes, extends, describes, and uses patterns based on arithmetic or geometric patterns (ex: predicts the number of colored regions in an expanding geometric pattern)  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Develops the general explicit term for a specified geometric pattern in symbols or words  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Uses the general term for a pattern to find specified terms in the pattern  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Translates among table, symbolic, graphical, and verbal descriptions for a relationship involving a variable quantity  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Identifies if the relationship describing the change in quantities is direct or inverse, i.e., linear or curved  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Develops symbolic and graphical representations for situations involving variable quantities (ex: finds an equation representing a given situation or drawing a graph of a specified relationship)  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Evaluates a symbolic algebraic expression for a specified value of the variable involved and simplifies expressions  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Develops an equation or inequality to represent a given problem situation and uses the equation to solve the problem  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Solves a linear equation or inequality and interprets the answer in the context giving rise to the equation or inequality  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X  
- Uses graphs, tables, or symbolic methods to solve, or estimate the solution of, a linear equation or system of linear equations  
  LINKS TO EALRs: 1.5  
  ASSESSMENT PROCESSES: X

**Key for Assessment Processes**

1. Illustrated journals
2. Focused observationianecdotal records
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## Suggested Assessment Evidence for the End of Tenth Grade (continued)

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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

### PROBLEM SOLVING

Divides complex problem into a number of smaller and easier problems to solve and identifies unknowns

| 2.1, 2.2 |

Systematically searches for patterns in complex situations

| 2.1 |

Identifies solution strategy based on the relationship of variables or patterns in problem situation

| 2.1, 2.2, 2.3 |

Collects and organizes information needed in order to solve open-ended problem

| 2.2, 2.3 |

Determines which materials, information, or strategies could be used in solving a given problem

| 2.3, 4.3 |

### MATHEMATICAL REASONING

Interprets evidence or information related to a problem to form a convincing argument related to a statement/generalization about the problem

| 3.1, 3.2, 3.3 |

Creates a counterexample to show invalid statements do not hold

| 3.1, 3.2 |

Demonstrates the reasonableness of an argument behind a given position or solution related to a problem, citing the evidence or information supporting the position taken

| 3.3 |

Defends the validity of a particular solution to a problem (ex: such as a detailing of a solution process and what the results mean)

| 3.2, 3.3, 4.3 |

### COMMUNICATION

Uses technology to gather and organize information related to problems

| 4.1, 4.2 |

Organizes and classifies the story told by data or information in light of known mathematics concepts and properties

| 4.2 |

Applies mathematical language and notation to represent and communicate findings to others (ex: such as through the use of graphs, charts, and tables)

| 4.2, 4.3 |

Relates mathematical concepts and procedures to problems based in everyday life

| 4.1, 4.2, 4.3, 5.3 |

### Key for Assessment Processes

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All of the Assessment Processes may be used for gathering information.
## Suggested Assessment Evidence for the End of Tenth Gr: CONNECTIONS

<table>
<thead>
<tr>
<th>LINKS TO EALRS</th>
<th>ASSESSMENT PROCESSES**</th>
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<td><strong>All of the Assessment Processes may be used for gathering information</strong></td>
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<td><strong>CONNECTIONS</strong></td>
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<td>Demonstrates an understanding of the relationships between ideas in mathematics, such as similarity and congruence in geometric settings</td>
<td>5.1</td>
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<td>Explains the connections between geometric representations of lines in the coordinate plane and their representations in symbolic algebraic form</td>
<td>5.1</td>
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<td>Creates and uses multiple representations for objects connecting mathematics with other disciplines concepts and connecting between areas of mathematics</td>
<td>5.1, 5.2</td>
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<td>Applies mathematical approaches to solving quantitative, spatial, or data-based problems in other content areas</td>
<td>5.2</td>
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<td>Uses mathematical relationships to tie the use of algebra and geometry together as problem-solving tools (ex: shifts geometric probability problem from a diagram to coordinate grid and uses algebra to solve the problem)</td>
<td>5.1</td>
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**Key for Assessment Processes**
1. Illustrated journals
2. Focused observationianecdotal records
3. Individual interviews
4. *Performance assessment*
5. Traditional paper-and-pencil tests
A Framework for Achieving the Essential Academic Learning

Requirements in Mathematics Grades 8-10 Glossary

**absolute value** the numerical value of a number without regard to its sign; the distance of the number from 0 on the number line. For example, the absolute value of 3 is 3, of -9 is 9, and of 0 is 0. The absolute value of -5 is written as \( | -5 | = 5 \).

**acute angle** an angle with measure less than 90 degrees and greater than 0.

**acute triangle** a triangle with three acute angles.

**adjacent angles** angles in the same plane that have a common side and a common vertex, but whose interiors do not intersect. For example, LACB is adjacent to LBCD.

**algorithm** a step-by-step method for computing. For example, the addition algorithm that describes how to find the sum when regrouping, or the long division algorithm.

**angle** two rays that share a common endpoint. We use the symbol \( \angle \) to indicate an angle. For example, see LCDE below.

**approximate** to give a number that is not exact but reasonably close.

**area** a measure in square units of the interior of a 2-dimensional figure.
arithmetic sequence  a list of numbers in which the difference between any two adjacent numbers is the same. The first number in the list is called the initial value. The list 1, 3, 5, 7, ... is an arithmetic sequence because the difference between any two adjacent numbers is 2. That difference is called the common difference.

associative property of addition  the sum stays the same when the grouping of addends is changed. That is, for all numbers in a given set, \( a + (b + c) = (a + b) + c \). For example: \( 4 + (45 + 8) = (4 + 45) + 8 \). This holds for all real numbers.

associative property of multiplication  the product stays the same when the grouping of factors is changed. That is, for all numbers in a given set, \( a(bc) = (ab)c \). For example, \( 4 \cdot (45 \cdot 8) = (4 \cdot 45) \cdot 8 \). This holds for all real numbers.

average  a measure of central tendency of a collection of data; usually expressed as the mean of the collection of data but includes median and mode.

axiom  a self evident and generally accepted statement. For example, two points determine a single straight line.

axis  see x-axis and y-axis. The plural of “axis” is “axes”.

box-and-whisker plot  a graph which displays the following five points from a data set: the minimum value, the lower quartile (25th percentile), the median, the upper quartile (75th percentile), and the maximum value. The rectangle represents the middle 50% of the data and the whiskers at both ends represent the remainder of the data and, along with the rectangle, show the spread of the data.

capacity  the measure of the amount of material or liquid that can be put in a container. For example, the number of ounces, the number of milliliters, and the number of gallons are measures of capacity.
central tendency a single number used as a typical value for a set of data. The mean, median, and mode are used as measures of central tendency.

circle the set of all points in a plane that are the same distance, called the radius, from a fixed point called the center of the circle.

circumference the distance around a circle. The circumference is the product $\pi d$ or $2\pi r$, where $d$ is the measure of the diameter and $r$ is the measure of the radius of the circle.

closure property a set of numbers is said to be closed under an operation if the result of performing the operation on any two numbers in the set produces in another number in the set.

coefficient the numerical part of an algebraic term. For example, 2 and 3 are coefficients in $2x + 3y$.

collinear points points on the same line.

commutative property of addition the effect of adding $a$ plus $b$ is the same as adding $b$ plus $a$. For example, the commutative property of addition states that for all real numbers $a$ and $b$, $a + b = b + a$.

commutative property of multiplication the effect of multiplying $a$ times $b$ is the same as multiplying $b$ times $a$. For example, the commutative property of multiplication states that for all real numbers $a$ and $b$, $a \cdot b = b \cdot a$.

coplanar points points that are on the same plane.

composite number an integer greater than 1 which has whole number factors other than itself and 1. For example, 10 is a composite number because it has the factors of 2 and 5, in addition to 1 and 10.

compound event an event that consists of two or more simple events. For example, consider the event of rolling a six on a number cube and flipping a coin with a result of tails.

conditional probability the probability that an event will occur given that another event has already occurred.
cone a 3-dimensional figure whose base is a circle and whose side is a curved surface that joins the base to a single point, the vertex.

cross-section a shape formed when a plane cuts through a 3-dimensional figure.

congruent figures figures that are exactly the same size and shape.

conjecture inference or judgment based on inconclusive or incomplete information.

coordinates an ordered pair of numbers that identify a point on a coordinate plane. For example, (3,4) is the coordinate of point A.
cube a regular polyhedron with six square faces.

cube $\,$ of a number the third power of a number, For example $4^3 = 4 \times 4 \times 4$.
cylinder a 3-dimensional figure having two parallel, congruent, circular bases and a surface connecting the bases.
decimal number a number expressed in base 10, such as 39.456.
deductive reasoning using logic, definitions, and other statements known to be true in order to prove general statements.
dependent event an event whose probability is determined by the outcome of another event.
derived measurement a measurement determined by finding the ratio of other measurements. For example, density is determined by dividing the mass of a quantity by its volume, speed by dividing distance covered by time elapsed.
direct measurement a measurement determined by the use of measuring instruments.
**distributive property**  the product of a number and a sum is equal to the sum of the products of the number with each of the addends in the sum. That is, for all real numbers \(a, b,\) and \(c\) in a given set, \(a(b + c) = ab + ac.\) For example, \(4(45 + 8) = (4 \cdot 45) + (4 \cdot 8)\).

**divisible**  one integer is divisible by another non-zero integer if the quotient is an integer with remainder of zero. For example, 12 is divisible by 3 because \(12 \div 3\) is an integer, namely 4.

**domain**  set of all values of the independent variable of a given function, usually the \(x\)-values on a coordinate plane.

**edge**  the line segment formed by the intersection of two faces of a 3-dimensional figure.

**equally likely**  two outcomes are equally likely if they have the same probability of occurring.

**equation**  a number sentence showing equality between two sets of values or expressions; contains an equals sign. For example, \(4 + 9 = 13\) or \(x + 5 = 3y - 9\).

**estimate**  a number close to an exact amount or the process resulting in such a value.

**event**  any subset of the sample space. In rolling a number cube, the event of rolling a “3” is a singleton event because it contains only 1 outcome. The event of rolling an “even number” contains 3 outcomes.

**experimental probability**  the ratio of the number of times an event occurs to the number of trials.

**exponent**  a number used to tell how many times a factor has been included in a specified product. For example, in \(x^3\), the exponent 3 indicates the term \(x\) appears three times as a factor.
expression

da symbolic representation of a quantity in terms of variables, numbers, and symbols. For example, \(2l + 2w\) is an expression for the perimeter of a rectangle of length \(l\) and width \(w\).

extrapolate
to estimate or approximate a value beyond a given set of data.

face

a polygon forming a side or a base of a 3-dimensional figure.

factor

one of the quantities being multiplied to form a product. For example, 2 and 3 are factors in \(2 \times 3\).

factorial

the product of all whole numbers from \(x\) down through 1, symbolized by \(x!\). For example \(3! = 3 \times 2 \times 1\), or 6.

fraction

a number expressing the number of parts present in a situation, given the number of parts that make a whole. For example, the fraction representing \(\frac{1}{3}\) of 3 parts is \(\frac{3}{8}\). A fraction can also be defined as a number of the form \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b \neq 0\).

generalization
to form conclusions through inductive reasoning—to infer from several cases.

geometric sequence

a sequence of numbers, called terms, in which each successive term is determined by multiplying the previous term by a common factor. For example, 1, 2, 4, 8, 16, ... is a geometric sequence with a first term of 1 and a common factor of 2.

graphical

a representation making use of graphs, drawings, or sketches.

greatest common factor (divisor)

the largest factor that is common to two or more terms.
histogram a bar graph that shows the frequency distribution for a set of data. The graph is noted for the labels of the bars being given in intervals and for no spaces between successive bars.

**Reaction Time to Drug Administration**

<table>
<thead>
<tr>
<th>Reaction Time (seconds)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>10</td>
</tr>
<tr>
<td>2-3</td>
<td>30</td>
</tr>
<tr>
<td>4-5</td>
<td>40</td>
</tr>
<tr>
<td>6-7</td>
<td>50</td>
</tr>
<tr>
<td>8-9</td>
<td>60</td>
</tr>
<tr>
<td>10-11</td>
<td>70</td>
</tr>
<tr>
<td>12+</td>
<td>80</td>
</tr>
</tbody>
</table>

**identity property for addition** for all real numbers $n$, there exists a number 0, such that $n + 0 = n$. That is, 0 is the additive identity; adding 0 to any number results in the same number.

**identity property for multiplication** for all real numbers $n$, there exists a number 1, such that $n \cdot 1 = n$. That is, 1 is the multiplicative identity; multiplying 1 by any other number results in the same number.

**impossible event** an event that cannot happen, or an event with a probability of 0.

**improper fraction** a fraction in which the numerator is larger than the denominator.

**independent events** two events whose outcomes have no effect on one another. For example, the outcome of the second flip of a coin is independent of the first flip of a coin.

**indirect measurement** a measurement determined without the direct application of measurement tools. For example finding a measure by the use of the Pythagorean theorem, by similarity, or through ratios or scale factors.

**inductive reasoning** a method of reasoning in which a conjecture is made based on several observations.

**inequality** any mathematical sentence that compares two expressions using one of the symbols $<$, $>$, $\geq$, $\leq$, or $\neq$. 
inference  a conjecture based on inductive reasoning.

integer  a number in the set of whole numbers and their additive inverses \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots \}

integral  refers to being an integer.

interpolate  to estimate or approximate a value between two given values.

inverse property of addition  each real number \(x\) has an additive inverse, denoted \(-x\), such that their sum is 0. For example \(-3\) is the inverse of 3 because \(-3 + 3 = 0\).

inverse property of multiplication  each non-zero real number \(x\) has a multiplicative inverse, denoted by \(\frac{1}{x}\), such that their product is 1. For example \(\frac{1}{3}\) is the multiplicative inverse of 3 because \(\frac{1}{3} \cdot 3 = 1\).

irrational number  a number that cannot be written in fraction form. For example, the number \(\pi\) cannot be written in fraction form.

least common multiple  the integer which is the least positive multiple of two or more specified positive integers. For example, 24 is the least common multiple of 6 and 8, as it is the smallest number that is a multiple of both.

length  a measure of the distance between two points along a line.

line of best fit  a line drawn on a scatter plot to estimate the relationship between two sets of data.

line graph  the graph of a first degree linear equation whose coordinates satisfy the equation and form a straight line.

linear equation  an equation whose graph on a coordinate grid is a line and that can be written in the form \(y = mx + b\).

linear inequality  an inequality whose graph on a coordinate grid is bounded by a line and that can be written in the form \(y \geq, <, >, \text{ or } \lt\) \(mx + b\).

line of symmetry  a line on which a figure can be folded into two parts that are congruent mirror images of each other.

mathematical system  a system consisting of a set of undefined terms, defined terms, basic statements-called axioms-whose truth is assumed, a system of logical reasoning, and a set of statements derived from them.
mean a measure of central tendency found by summing all members in a set of data and dividing by the number of members in the set. For example, the mean of \( \{3, 4, 8\} \) is 5. The mean is often called the average.

measurement the numerical amount associated with dimensions, quantity, length, or capacity. For example, the length of a line segment or the volume of a cube are measurements.

median the number in the middle of a set of data arranged in order from least to greatest. For example, 7 is the median of \{2, 4, 7, 8, 9\}. If there are an even number of elements in the set, the median is the mean of the two middle numbers.

mixed number a numeral consisting of a whole number and a fraction, such as \(2 \frac{3}{4}\).

mode the item than occurs most frequently in a set of data. There may be one, more than one, or no mode. For example, the mode in \{1, 3, 4, 5, 5, 7, 9\} is 5.

multiple any number that is the product of a number and an integer. For example, 24 is a multiple of 3, since \(24 = 3 \times 8\).

natural number a number from the set of numbers \{1, 2, 3, 4, \ldots\}. The natural numbers are the positive integers.

noncoplanar points a set of points in space that cannot be contained in the same plane.

nonrepeating decimal a decimal number that does not have a repeating sequence of non-zero digits. For example, \(x = 3.14159\ldots\).

number line a line that shows numbers ordered by magnitude from left to right or bottom to top; an arrowhead at each end indicates that the line continues endlessly in both directions.

obtuse angle an angle with measure greater than 90 degrees and less than 180 degrees. \(\angle ABC\) is an obtuse angle.
obtuse triangle a triangle with one obtuse angle.

open-ended problem a problem with different possible solution paths and which may have different solutions depending on the route taken.

order of operations in simplifying an expression involving a number of indicated operations, one performs the operations in the following order:
1. completes all operations inside parentheses first;
2. finds powers and roots and in the order they occur from left to right;
3. performs all multiplication and divisions-left to right
4. performs all additions and subtractions-left to right

For example, the expression \((3' + 3)\cdot 2 - (5 - 3) + 2\) can be simplified by first changing the expression in the following ways:
\[
\begin{align*}
(3^2 + 3)\cdot 2 - (5 - 3) + 2 \\
(9 + 3)\cdot 2 - (5 - 3) + 2 \\
(12)\cdot 2 - (2) + 2 \\
24 - 1 \\
23.
\end{align*}
\]

ordered pair a pair of numbers that gives the location of a point on a coordinate grid in the order of (horizontal coordinate, vertical coordinate); often expressed as \((x,y)\).

outcome one of the possible results in a probability situation or activity.

outlier a number in a set of numbers that is significantly smaller or larger than most of the other numbers in a set.

parallel lines lines that lie in the same plane and never intersect; they are the same distance apart at all points.

parallelogram a quadrilateral with opposite sides parallel.

percent a way of expressing a ratio to compare a number to 100. The first number in such a comparison is the percent. For example, \(75:100 = 75\%\).

perimeter of polygon the sum of the lengths of the sides of the polygon.

perpendicular lines lines that lie on the same plane that intersect to form right angles (90 degrees). For example, lines \(l\) and \(m\) are perpendicular lines.
pi

the ratio of the circumference to the diameter of the same circle. The value of pi is approximately 3.14159 and pi is represented by the symbol π.

polygon

a closed plane figure formed by three or more straight segment sides whose only intersection is at their vertices. For example, ABCDEF is a polygon.

polyhedron

a closed 3-dimensional figure in which all of the surfaces are polygons. For example, ABCD is a polyhedron.

power

a term of the form \( x^n \) resulting from repeated multiplication of a factor. For example, 16 or \( 2^4 \) is the fourth power of 2, since 2 has been used as a factor four times.

predict

to state, tell about, or make known in advance, especially on the basis of special knowledge.

prime number

a whole number greater than 1 having exactly two whole number factors, itself and one. For example, 7 is prime since its only whole number factors are 1 and 7.
**prism**  
A 3-dimensional figure with 2 congruent and parallel polygonal bases, so that all other faces are parallelograms.

**probability**  
A measure of the chance that an event will occur; the ratio of the number of favorable outcomes to the number of possible outcomes.

**proof**  
A logical argument that a specified statement is true based on assumed statements and previously determined true statements.

**proportion**  
An equation showing two ratios are equal. For example, \( \frac{1}{6} = \frac{5}{x} \) is a proportion. It can be written \( 3:4 = 5:x \).

**pyramid**  
A 3-dimensional figure whose base is a polygon and whose sides are triangles that meet at a common point, the vertex. The shape of the base names a pyramid. The example below is a rectangular pyramid, because its base is a rectangle.

**Pythagorean theorem**  
In any right triangle having a hypotenuse of length \( c \) and two legs of lengths \( a \) and \( b \), \( a^2 + b^2 = c^2 \).

**quadratic equation**  
An equation of the form \( y = ax^2 + bx + c \) where \( a \neq 0 \).
quadrilateral  
a polygon with four sides. For example, ABCD is a quadrilateral.

range (statistical)  
the absolute value of the difference between the largest and smallest values in a set of data. For example, the range of \{2, 4, 6, 7, 9, 13\} is 13 – 2 or 11.

range (functional)  
the set of all values of the dependent variable of a given function, usually the y-value on a coordinate plane.

rate  
a ratio comparing two quantities measured in different units where one is measured in time. For example, miles per hour and heartbeats per minute are rates.

ratio  
a comparison of two numbers using division. For example, two to three can be expressed as a ratio as \(\frac{2}{3}\) or 2:3.

rational number  
a number that can be expressed in the form \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b \neq 0\) (see fraction).

real number  
any number that can be expressed in decimal form.

reciprocal  
the multiplicative inverse of a non-zero number. For example, the reciprocal of \(x\) is given by \(\frac{1}{x}\). The reciprocal of -3 is \(-\frac{1}{3}\).

rectangle  
a quadrilateral with four right angles:
rectangular prism  a polyhedron with six rectangular faces. For example, the figure shown is a rectangular prism.

reflection on a line  a transformation of a figure by reflecting it over a line, creating a mirror image of the original figure.

reflection on a point  a transformation of a figure by reflecting each of its points through a fixed point, called the center of the reflection, creating an image of the original figure across the center.

regular polygon  a polygon with all sides having the same length and all angles having the same measure. For example, ABCDEF is a regular polygon called a hexagon.
regular polyhedron  a polyhedron with congruent regular polygons for all faces

repeating decimal  a decimal number whose expression contains a repeating pattern of decimals from some point in the expression forward. For example, 3.121212… is a repeating decimal with the repeating pattern of the digits “12”. This decimal can be written as $3.\overline{12}$.

rhombus  a parallelogram with four congruent sides. For example, $ABCD$ is a rhombus.

rotation  is a transformation of the points $P$ in a plane resulting from turning a figure about a specified point $O$ a fixed number of degrees or fractional portion of a turn either clockwise or counterclockwise.

sample space  the set of all possible outcomes to a specified event.

scale  sequenced collinear marks, usually at regular intervals or else representing equal steps, that are used as a reference in making measurements.

scale factor  a ratio that compares two sets of measurements such as the size of a model to the actual size of the object being modeled.

scatter plot  a graph of points $(x, y)$, one for each item being measured, on a coordinate plane. The two coordinates of a point represent their observed, paired values. For example, the ordered pairs may relate temperature to time of day.
**scientific notation**  a number expressed in the form $a \times 10^n$ where $1 \leq a < 10$ and $n$ is an integer. For example, 342.15 can be written in scientific notation as $3.4215 \times 10^2$.

**sequence**  an ordered list of objects, especially numbers, whose elements are called terms. For example, $2, 4, 6, 8, \ldots$ is the sequence of positive even numbers.

**series**  the indicated sum or difference of a sequence of numbers. For example, the series $1 + 3 + 5 + \ldots + 9$ is the series of the first 5 odd whole numbers.

**similar figures**  two geometric figures that have the same shape, but not necessarily the same size. The measures of the corresponding sides are proportional and the measures of their corresponding angles are equal. For example, $ABCD$ and $EFGH$ are similar figures.

**simple fraction**  a fraction whose numerator is an integer of lower absolute value than its denominator with no common factors except 1.

**simulation (probability)**  using an experiment based on a real-life situation to answer a question. For example, toss a coin to simulate true-false; heads=true, tails = false.

**slope**  the ratio of the change in $y$-units (vertical) to the change in $x$-units (horizontal) between two points on a line. For example, the slope of a line through $(3,4)$ and $(9,5)$ is $\frac{5-4}{9-3}$ or $\frac{1}{2}$.

**solution**  a number that, when substituted for the variable in an equation, results in a true statement.

**solve**  the process, or procedure, used to find the solution to an equation or problem.

**spatial**  of, pertaining to, involving, or having the nature of space.
**sphere**

the set of all points in 3-dimensional space that are at a fixed distance, called a radius, from a point called the center.

![Sphere Diagram](image)

**square**

a rectangle with four congruent sides. For example, ABCD is a square.

![Square Diagram](image)

**square number**

an integer that is a perfect square of another integer. For example, 49 is the square of 7. That is, a product of a number multiplied by itself.

**square root**

one of two equal non-negative factors of a given number. For example, 7 is the square root of 49 because \(7 \cdot 7 = 49\).

**successive events**

events that follow one another in a compound probability setting.

**surface area**

the sum of the areas of all the faces of a 3-dimensional object.

**symbol**

a letter or sign used to represent a number, function, variable, operation, quantity, or relationship. For example, \(a, =, +\).

**symmetry (line)**

the geometric property of being balanced about a line. For example, a figure is symmetric with respect to a line, called the axis of symmetry, if it can be folded on the line and the two halves of the figure are congruent and match.
symmetry (point) a figure is symmetric about a point if there is a rotation of the figure of less than 360 degrees about the point that allows the figure to correspond with itself.

system of equations two or more equations in terms of the same variables. The solution of a system is a set of values for the unknowns (variables) that satisfies all the equations simultaneously.

tabular organized in a table or list.

terminating decimal a decimal that contains a finite number of non-zero digits.

theoretical probability a measure of the likelihood that an event will occur; is equal to the ratio of favorable outcomes to the number of possible outcomes. For example knowing that there are six possible outcomes for rolling a fair number cube, one can assign the probability of 1/6 to each of the possible outcomes.

transformation (geometric) a change in position of a figure using a translation, reflection, rotation, or combinations of these mappings.

translation a transformation where every point of a figure moves the same distance in the same direction.

trend a functional relationship between observed data and an independent variable. A conclusion reached by using the line of best fit.

triangle the figure formed by joining three non-collinear points with straight segments.

undefined term a term whose meaning is not defined in terms of other mathematical words, but instead is accepted with an intuitive understanding of what the term represents. For example, the words “point,” “line,” and “plane” are undefined terms from geometry.

unique indicates when there is one and only one object or result. For example, the product of two integers is unique.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>valid statement</td>
<td>a statement taken as being true in a reasoning situation.</td>
</tr>
<tr>
<td>validate</td>
<td>the process of determining, substantiating, verifying, or confirming whether a given statement or argument passes the standards for correct reasoning.</td>
</tr>
<tr>
<td>variability of data</td>
<td>range, average deviation, standard deviation, and spread are all ways of describing the variability of data.</td>
</tr>
<tr>
<td>variable</td>
<td>a symbol used to represent a quantity that changes or can have different values. For example in $5n$, the $n$ is a variable.</td>
</tr>
<tr>
<td>variation (direct)</td>
<td>a relationship between two variables that can be expressed in the form $y = kx$ where $k \neq 0$. $y = kx$ can be read as “$y$ varies directly with respect to $x$.”</td>
</tr>
<tr>
<td>variation (inverse)</td>
<td>a relationship between two variables that can be expressed in the form $y = \frac{1}{k}$ where $k \neq 0$. $y = \frac{1}{k}$ can be read as “$y$ varies inversely with respect to $x$.”</td>
</tr>
<tr>
<td>vertex</td>
<td>a point at which 2 lines meet to form an angle, where edges of a polygon or polyhedron intersect, or the point opposite the base in a pyramid or cone.</td>
</tr>
<tr>
<td>volume</td>
<td>a measure in cubic units of the space contained in the interior of a solid figure. For example, the number of cubic units contained in the interior of a rectangular solid.</td>
</tr>
<tr>
<td>whole number</td>
<td>a number from the set of numbers ${0, 1, 2, 3, 4,\ldots}$. The whole numbers are the non-negative integers.</td>
</tr>
<tr>
<td>x-axis</td>
<td>one of two intersecting straight lines that determine a coordinate system in a plane; typically the horizontal axis.</td>
</tr>
<tr>
<td>y-axis</td>
<td>one of two intersecting straight lines that determine a coordinate system in a plane; typically the vertical axis.</td>
</tr>
</tbody>
</table>